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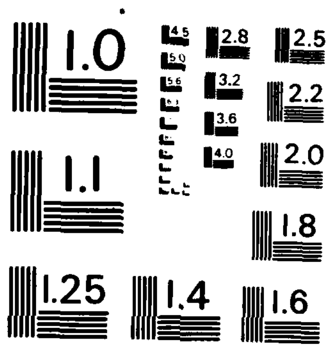

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CALIBRATION OF HOLE-PRESSURE MEASUREMENTS IN NON-NEWTONIAN FLOW BY NUMERICAL METHODS

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ABSTRACT

The crossed-triangle macroelement has been identified as an ideal element for non-Newtonian fluid flow calculations involving fluids with integral constitutive equations. In spite of an instability of the pressure approximation, these elements have been shown to have convergent velocities in Stokes flow, and there is strong evidence that pressure-smoothing schemes recover accurate pressures. Earlier studies by the author of Newtonian flow over transverse slots at low Reynolds numbers showed that excellent results could be obtained using the element. Studies of non-Newtonian flows in the same geometry showed good qualitative agreement with laboratory experiment but led to some puzzling predictions of the pressure difference between the top and bottom of the slot (the "hole-pressure"). In this paper, those puzzling predictions are re-examined, and the deviations from expectation are re-interpreted. They appear to make physical sense and have important ramifications for the calibration of devices which measure the primary normal-stress difference by continuous measurement of the hole-pressure.

AMS (MOS) Subject Classifications: 65N30, 76A05, 76A10

Key Words: non-Newtonian fluid, integral constitutive equation, crossed-triangle macroelement, hole-pressure, normal-stress difference, streamline coordinate system

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CALIBRATION OF HOLE-PRESSURE MEASUREMENTS IN NON-NEWTONIAN FLOW BY NUMERICAL METHODS

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INTRODUCTION

In refs. [1 - 3], the linear crossed-triangle macroelement is described, and some calculations using it are reported. In refs. [1] and [2], results on the pressure difference between the top and bottom of a transverse slot in plane Newtonian flow are presented. It was found that the element accurately reproduced the value of the slope of the curve obtained when pressure difference, divided by undisturbed wall shear stress, is plotted against hole-based Reynolds number [1 - 3] in the low Reynolds number regime, where the slope is known to be approximately -1/30 [4]. The value reported in refs. [1] and [2] obtained using a fairly crude mesh was -0.031. Subsequent mesh refinement and grading of the mesh have produced a value of -0.033. All indications are the crossed-triangle element produces excellent results in Newtonian flows.

It was with some puzzlement, then, that some of the results of refs. [1] and [3] were reported. These results involve the pressure difference over a transverse slot in non-Newtonian flow. A relation (given below) has been proposed relating this difference to the primary normal-stress difference, N_1, a very important measure of the fluid's viscoelasticity. The numerical results of refs. [1] and [3] indicate strongly that the assumptions behind the derivation of the pressure difference/normal-stress difference relation (the "HPBL relation" [3]) are violated by non-Newtonian flows at significant non-dimensional shear rates (the "Deborah number" [1]). Nevertheless, what seem to be extremely carefully conducted laboratory experiments [5], [6] suggest that the quantitative prediction of the HPBL relation is correct, in spite of the inapplicability of the derivation. Unfortunately, the numerical experiments reported in refs. [1] and [3] do not agree with the laboratory experiments, at least at first sight. The pressure differences from the numerical model are systematically lower than the HPBL prediction.

The author has some confidence in his numerical prediction of non-Newtonian pressure differences, however. The systematic overestimation of pressure differences by the HPBL relation has also been observed numerically by Dupont, Marchal, and Crochet [7], and the results of ref. [7] can be very accurately duplicated by this author's own code [8]. The author's code and that of ref. [7] are quite different implementations of the same sort of approach to memory fluid problems. The author's code uses the crossed-triangle elements, whereas ref. [7] employs eight-node rectangular elements.

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In this paper we offer a new interpretation of the numerical non-Newtonian hole-pressure predictions. This interpretation results in a simple calibration rule for the HPBL relation. The calibration rule leads to a modified HPBL prediction which seems to correspond to a reasonable modification of the derivation of the HPBL relation which corrects some of its observed inadequacy. At this point, we can only conjecture why the laboratory experiments do not require the calibration the numerical experiments require, but an intriguing possiblity is that deformable membrane transducers may not measure pressures at the points one might expect that they do, and that the deviation of the effective measurement point of such a transducer may amount to a fortuitous self-calibration.

PRELIMINARIES

The emphasis of this paper is on the interpretation of numerical results and on the extent to which the physical reasonability of those results reflect upon the performance of the crossed-triangle macroelement. The details of the finite element formulation and the physical idealizations involved in the slot-flow problem are described amply elsewhere. The purpose of this section is to remind the reader of several points and direct him/her to sources where further details may be found.

Flows over transverse slots

The slot-flow problem discussed here is described in most complete detail in ref. [3]. The notation and other conventions described there are adopted here. Figure 1 shows the idealized domain of the problem and its dimensions.

This problem is also discussed in refs. [1,2,4 — 10]. There are several important definitions and equations which will bear repeating here. First we define a Reynolds number for a shearing flow,

$$R_L(\dot{\gamma}) = \frac{\rho h b \dot{\gamma}}{4\mu(\dot{\gamma})} \tag{1}$$

where ρ is the fluid density ($\rho = O(1)$ for fluids considered here), h is the height of the channel into which the slot is cut, and b is the width of the rectangular slot. The slot is taken to be at least three times deeper than it is wide. $\dot{\gamma}$ is the shear rate and $\mu(\dot{\gamma})$ the viscosity. We are concerned here only with the case $R_L(\dot{\gamma}) \approx 0$. This is assured in the present study by modelling very viscous fluids in which $\mu(\dot{\gamma})$ is large enough to keep $R_L \leq O(10^{-2})$ at shear rates $\dot{\gamma} \leq O(10^1)\ s^{-1}$. The fluids being modelled here may be thought of as polymer melts or highly concentrated polymer solutions.

When $R_L \approx 0$, the HPBL relation has an integral form [3]

$$P_e = \frac{1}{2}\int_0^{\sigma} \frac{N_1}{\tau}\, d\tau \tag{2}$$

The pressure difference between centerline measurement points at the bottom of the slot and on the channel wall opposing the slot mouth is predicted by one-half the integral of the stress ratio as a function of shear stress, from zero to the value σ, of the wall shear stress in the flow in which P_e is to be observed. The values of $N_1(\tau)$ are values observed in a simple shearing flow which produces a corresponding shear stress, τ. Thus the HPBL

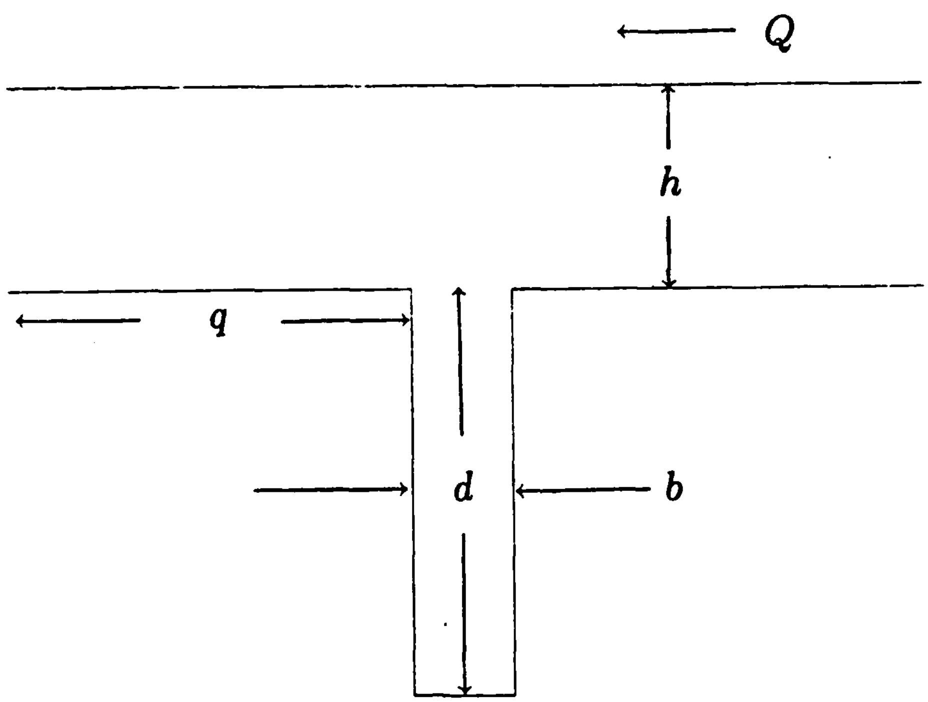

FIGURE 1

Domain for idealized slot-flow problem: cross-section of wide channel of depth h with slot of width b and depth d; q is large with respect to b and h. Q indicates direction of flow.

prediction of Eq. (2) predicts P_e in slot flow based on an integral of viscometric functions. Differentiation of Eq. (2) with respect to σ yields the measurement relation

$$N_1 = 2\frac{dlnP_e}{dln\sigma}P_e \tag{3}$$

which enables viscometric values of N_1 to be predicted by continuous measurement of hole-pressure [1,3,5,6].

The values of σ in Eqs. (2) and (3) are nominally the values of σ in undisturbed channel flow, because this value of σ can be reliably measured by streamwise pressure differences [1,3 - 5]. However, if $h/b \ll 1$, the disturbance in the channel flow induced by the slot is significant at the wall opposing the slot [1], resulting in values of shear stress and first normal-stress difference $\sigma^0 < \sigma$ and $N_1^0 = N_1(\sigma^0) < N_1(\sigma)$ at the centerline opposite the slot mouth. In all of the results presented here, $h/b = 2$, so that $\sigma^0 \approx \sigma$ and $N_1^0 \approx N_1(\sigma)$. In all calculations reported here σ^0 is used in place of σ in Eq. (2), though either σ or σ^0 could be used without significantly altering the conclusions. Inspection of the derivation of the HPBL relation [10] indicates that σ^0 is indeed the correct value of the shear stress to use in Eq. (2).

It is also useful to characterize the degree of nonlinearity in a given flow. To do this, we will use the Deborah number,

$$D_e \equiv T\dot{\gamma} \tag{4}$$

where T is the dominant relaxation time of the fluid, and $\dot{\gamma}$ is the shear-rate at the wall in undisturbed channel flow. D_e may be thought of as a non-dimensional shear rate. Following ref. [7], we will find that a non-dimensional shear stress is useful:

$$S \equiv \frac{T\sigma}{\mu(0)} \tag{5}$$

where $\mu(0)$ is the viscosity at zero shear. It is often useful to consider S evaluated at the centerline of the slot on the wall opposing it,

$$S_0 \equiv \frac{T\sigma^0}{\mu(0)} \tag{6}$$

It should be noted that for fluids which do not have a shear thinning viscosity, D_e and S are equal. For shear-thinning fluids, S is always smaller than D_e, and may be significantly so. The thinning ratio, S/D_e, gives a measure of the amount of shear thinning at shear rate $\dot{\gamma}$, when $\sigma = \sigma(\dot{\gamma})$.

Constitutive Equations

Ref. [8] describes the general form of the constitutive equations employed to obtain the results reported here. Ref. [9] gives the details of the specific constitutive equations. In all cases we have taken

$$\begin{aligned} \mu(0) &= 410.125 \text{ Pa - s} \\ T &= 0.9666 \text{ s} \end{aligned} \tag{7}$$

and used: Curtiss-Bird with $\epsilon = 0.375$ and $\Lambda = 0$ (see Example 26, p. 6 of ref. [9], also refs. [1] and [3]), and what is referred to in ref. [9] as "the concocted constitutive equation" with $\Lambda = 0.3222$, $a = 0.87$, $c = 0.1$ (see Example 3b, p. 7 of ref. [9], modified as described on p. 23). This modification of the Johnson-Segalman constitutive equation was made to imitate the Phan Tien-Tanner model (see Example 1a, p. 6 of ref. [9]), which has a bounded extensional viscosity when $\epsilon \neq 0$, as does the model discussed here when the corresponding parameter, $c \neq 0$. The modified Johnson-Segalman model is obtained by formally adding a Wagner-like damping function, and so it will be referred to as the JSW model here. The Curtiss-Bird model will be referred to as the CB model hereafter.

Finite Element Approximation

The most complete description of the finite element method for memory fluids can be found in ref. [1], and it is also discussed in refs. [3,8, and 9]. The mesh employed is the 1008 macroelement mesh pictured in ref. [9]. The mesh is graded in the neighborhood of the singularities at the slot corners. The order of these singularities is known for Newtonian flow [1], but they are unknown in non-Newtonian flows. The crossed-triangle macroelement has an inherently unstable pressure approximation [2]. On many meshes, this does not seem

to affect the solution pressures, but in the problem at hand it definitely does. Raw pressures are severely checkerboarded in the non-Newtonian case, as they are in the Newtonian case [1,2]. The non-Newtonian pressures seem to be improved by the pressure smoothing scheme given in ref. [1] to the same extent the Newtonian pressures are. Since the publication of ref. [1], the pressure smoothing scheme has been rewritten to apply to all stress and strain-rate components, whether or not they involve a pressure contribution. All values of stress and strain rate reported here are smoothed values. This has the advantage of referring all stresses and strain rates to macro corner nodes, and eliminates the need to worry about making sure that stresses and pressure differences are referred to equivalent spatial locations [1].

Time-History Quadrature

Here as in refs. [1,3, and 7], P_e is predicted by integration of the viscometric function, $N_1(\tau)/\tau$, with the same time history quadrature [1,3] as was used to solve the slot flow problem. Since the writing of refs. [1,3, and 7], it was found that the 10-point quadrature [3] used for the history integral does not evaluate the viscometric functions accurately at the high shear rates which it is now possible to achieve using some constitutive equations. [The viscometric functions are usually known analytically or very accurately.] For the results reported here, 18-point formulas were generated and used for Curtiss-Bird. A tabulated 16-point Laguerre formula was used for the constitutive equations with a single-exponential memory.

For some constitutive equations – Curtiss-Bird, in particular – even 18 points do not seem sufficient at high D_e. At about $D_e = 15$, there begins to be visible inaccuracy in plots of $N_1(\tau)/\tau$ vs. τ. We present results there nevertheless, because they represent a perfectly valid test of the HPBL relation, with constitutive equation modified from the Curtiss-Bird by quadrature error. So long as we use the same quadrature for slot flow as the viscometric functions, the HPBL relation should hold if it is valid, since it proports to be independent of constitutive equation.

NUMERICAL RESULTS

The results we present here should be viewed as results in the same vein as those presented in refs. [1] and [3], but with an improved mesh, improved time history quadrature, and another constitutive equation. In refs. [1,3, and 7], the value of P_e is predicted by integrating Eq. (2) numerically from viscometric data. For the results presented here we have done the same and used the trapezoidal rule with 200 points, which tests showed to be quite sufficient. In refs. [1,3, and 7] the predicted P_e was plotted on the same curve as the actual P_e taken from the finite element model. The abscissa was σ, or more precisely σ''. Here we have chosen to plot P_e/N_1 vs. S_0. The results are summarized in Figure 2.

The systematic deviation of the actual hole-pressure from the predicted is reflected in the departure of the trend of the points below the "Tanner-Pipkin line", $P_e/N_1 = 0.25$. Tanner and Pipkin [11] presented an argument which implies that, at least asymptotically as $S_0 \to 0$, $P_e/N_1 \to 0.25$. According to the HPBL relation, any constitutive equation for which $N_1(\tau)/\tau$ is linear should have $P_e/N_1 = 0.25$, whether or not the flow is slow.

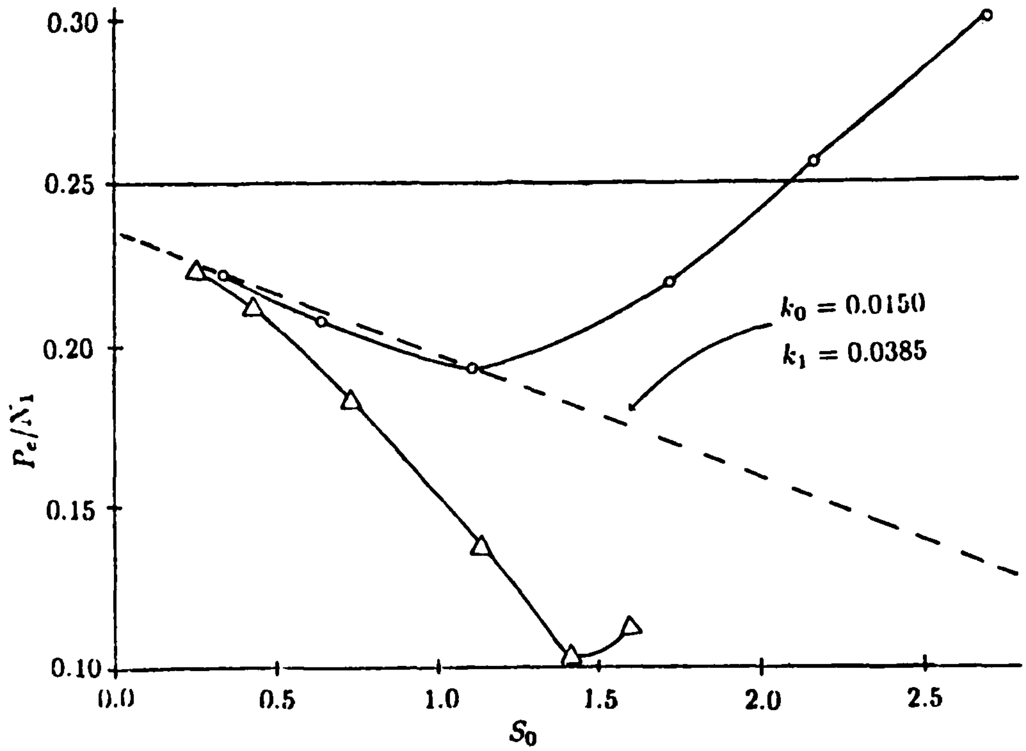

FIGURE 2

Numerically observed P_e/N_1 for JSW (○) and CB (△) models, plotted vs. S_0. Line at $P_e/N_1 = 0.25$ is Tanner-Pipkin line; dashed line is k-line, with intercept $0.25 - k_0$ and slope k_1.

For the constitutive equations considered here, $N_1(\tau)/\tau$ is very near linear in the range $0 \leq S_0 \leq 0.5$, but P_e/N_1 departs quite significantly from 0.25 in that region.

It is not clear from ref. [7] how well the Tanner-Pipkin limit is recovered, but it appears that those authors actually get a number slightly larger than 0.25, though it is difficult to precisely interpret their figure in that range. Until recently, the author believed that the failure of the numerical results to hit the Tanner-Pipkin line, at least asymptotically when $S_0 \approx 0$, was due to discretization error. The author's results at very low S_0 do get closer to the Tanner-Pipkin line with mesh refinement (more precisely, on the cruder mesh of refs. [1] and [3], the results are even farther away from the Tanner-Pipkin line than those given here). Also, the work of Webster [12] using a finite difference method, the Maxwell fluid (see Example 3b, p. 7 of ref. [9] with $\Lambda = 0$ and $a = 0$), and the 'Oldroyd B' fluid (same as Example 3b, p. 7 of ref. [9] with $\Lambda \neq 0$ and $a = 0$) shows values nearly on the Tanner-Pipkin line. Webster and this author are currently collaborating in a re-investigation of these low S_0 results, and Webster's latest results agree more with those presented here [13]. His results on a refined mesh no longer hit the Tanner-Pipkin line, but tend to a value somewhat below it for very small S_0. This departure from the Tanner-Pipkin result cannot be explained at present, but, for the sake of arguement, let us take it at face value here. What is observed in Figure 2 is that the low S_0 results fall

near a line with a negative slope and intercept. This line was obtained by interpolating linearly to the first and third JSW values, which should have fallen nearly precisely on the Tanner-Pipkin line, if the HPBL hypothesis had held. We observe that those values from the CB model which sould be very near the Tanner-Pipkin line fall very near the interpolating line, which we shall hereafter refer to as the "k-line." The difference between 0.25 and the k-line values thus represents a set of values which correct the low S_0 values from both the CB and JSW results when added to them. They shift the low S_0 results from their observed position to the Tanner-Pipkin line, where they "should be," if the HPBL hypothesis were correct.

This procedure corrects the lower S_0 results of Figure 2, which depart from the HPBL prediction, since $N_1(\tau)/\tau$ is linear, but P_e/N_1 differs significantly from 0.25. However, there is a significant deviation of the numerical results from the HPBL prediction in the higher S_0 range where $N_1(\tau)/\tau$ is no longer linear. This is illustrated in Figure 3 for the JSW model and in Figure 4 for the CB model.

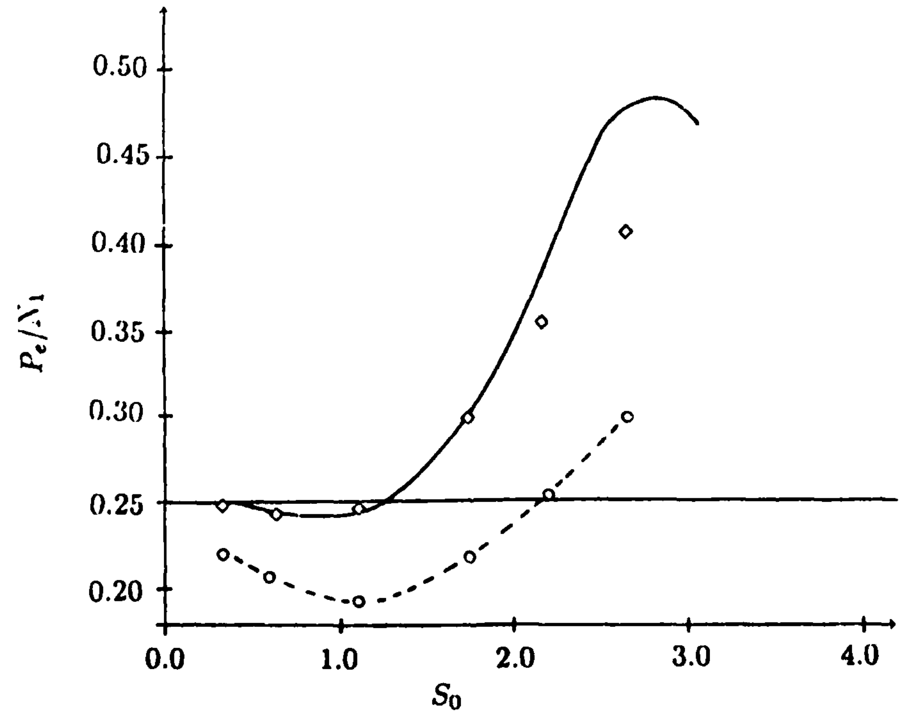

FIGURE 3

JSW predictions (solid line) and numerically observed (○) P_e/N_1 vs. S_0. Diamonds are calibrated values.

It should be noted that the upturn in the CB prediction curve at higher S_0 is known to be artificial and is an artifact of the Gaussian quadrature, as mentioned earlier. The downturn in the JSW prediction at high S_0 is believed to have the same cause. Here, we shall treat the inaccurately integrated constitutive equations as valid equations with

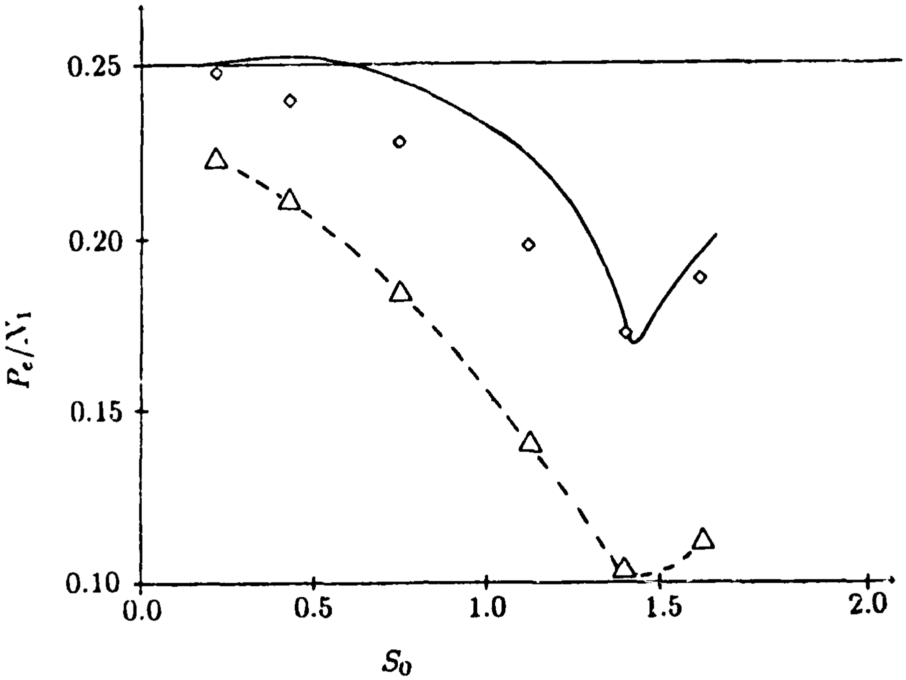

FIGURE 4

CB predictions (solid line) and numerically observed (△) P_e/N_1 vs. S_0. Diamonds are calibrated values.

viscometric functions which differ from JSW and CB somewhat at the higher S_0. For the purposes of testing the HPBL relation, the upturn and downturn are "real" enough. There are two important aspects of the observed and predicted values of P_e/N_1 which bear on what follows: First, the predicted and observed curves have the same gross, qualitative features; the CB observations even reflect the upturn of the prediction curve. Second, the observation curves behave with respect to the k-line the way the prediction curves behave with respect to the Tanner-Pipkin line, at least to a good approximation (but not exactly, because the observation curves would cross each other ever so slightly or be closer than they are, near $S_0 = 0.5$). However, the departure of the the observations from predictions are so quantitatively severe that the precision of the HBPL relation is certainly called into question.

A CALIBRATION RULE

The more empirical and less understandable statement of the calibration rule is "substitute the k-line for the Tanner-Pipkin line", or quantitatively

$$P_e = -k_0 N_1 - k_1 N_1 S_0 + \frac{1}{2}\int_0^{\sigma^0} \frac{N_1}{\tau}\, d\tau \tag{8}$$

where k_0 is 0.25 minus the intercept of the k-line, and k_1 is the negative of the slope of the k-line. There does not seem to be any physical interpretation readily apparent for Eq. (8). But a rearrangement of Eq. (8) does seem to make physical sense.

Let us assume that the flow at the wall opposite the slot is essentially a flow with constant pressure gradient, P_x. Numerical results verify the virtual constancy of P_x, even when $h/b = 1$. Let x be the streamwise coordinate which is zero at the slot centerline. At the wall, the thrust determining P_e is

$$P(x) = xP_x + P(0) \tag{9}$$

Letting

$$P_e = P_1 - P_2 \tag{10}$$

where $P_1 = P(0)$ and P_2 is the thrust at the bottom center of the slot. Then from Eqs. (10) and (11)

$$P(x) - P_2 = xP_x + P_e \tag{11}$$

For any choice of x,

$$P_e' \equiv P(x) - P_2 \tag{12}$$

is a modified pressure difference. Furthermore, if we compare Eqs. (9) and (12) we find that

$$P_e' = \frac{1}{2}\int_0^{\sigma^0} \frac{N_1}{\tau}\, d\tau \tag{13}$$

whenever

$$xP_x = k_0 N_1 + k_1 S_0 N_1$$

P_x is determined by the Poiseuille flow up and downstream of the slot, thus $P_x = 2\sigma/h$ and

$$x = k_0 \frac{N_1}{2\sigma} + \frac{k_1 h}{2\sigma} S_0 N_1 \tag{14}$$

This can finally be simplified and rewritten it terms of non-dimensionalized viscometric functions and constants with units of length:

$$\begin{aligned} x &= C_1\left(\frac{N_1}{2\sigma}\right) + C_2\left(\frac{TN_1}{2\mu(0)}\right) \\ C_1 &= k_0 h \\ C_2 &= k_1 h \end{aligned} \tag{15}$$

wherein the approximation $\sigma'' \approx \sigma$ has been made; this essentially amounts to an assumption that $h/b \geq 1$. With this same assumption, then, the calibration rule states that the P_e' computed using x determined according to eq. (15), satisfies the original HPBL hypothesis of eq. (2). We note that values of k_0 and k_1 determined by numerical modelling have always turned out to be positive (corresponding to a negative slope of the k-line and an intercept below the Tanner-Pipkin line). Thus the point, x, at which the upper pressure measurement must be made in order to satisfy the HPBL hypothesis is found to the upstream side of the slot centerline.

The reader unfamiliar with ref. [10] may find the second interpretation of the calibration rule as arcane as the first, but the author believes that it is not. The derivation of ref. [10] applies to flows for which, to an acceptable order of approximation, the streamlines and stress distribution are symmetric about the centerline of the slot. Though this is clearly not true for elastic fluids at high enough D_e, the streamlines and their orthogonal curves always form an orthogonal, curvilinear coordinate system, and if symmetry holds to sufficient accuracy, the centerline is then a coordinate curve on which the flow is essentially a simple shearing flow. The HPBL equation (2) follows from integrating along that coordinate curve. However, it is observed both numerically and experimentally that at higher D_e, the asymmetry about the centerline is severe [9]. In fact, the observed tilt of the vortex in the slot is such that the orthogonal curve which coincides with the slot centerline below the top vortex and passes through its center, is deformed so that it arrives at the wall opposing the slot at a point upstream of the slot centerline. Numerical and experimental results show there may actually be several vortices below the top one, which is near the slot mouth, but those vortices below the top one are so faint, when they occur, that the orthogonal curves are not visibly perturbed, and the flow is essentially still in the slot below the top vortex. This is illustrated by schematic in Figure 5.

What the author believes the calibration rule is, then, is evidence that there exists a modified HPBL relation which is referred to a path of integration from the bottom of the slot to the wall opposing the slot, ending up at a position upstream of the slot centerline. This coordinate curve connects the bottom of the slot to the center of the top vortex, passes up through the channel, and arrives at the top wall at approximately at the point identified by eq. (15).

That such a modified path could exist under reasonable assumptions about the flow, and the integration of ref. [10] could be carried out on a path which is not axially aligned and lead to the same result as the HPBL analysis, is still a very much unresolved issue requiring a careful analytic approach to the problem. Such an analysis should also attempt to identify the orders of approximations involved in the assumptions required to make the analysis go through, since the assumptions are not likely to ever be satisfied exactly in an elastic fluid. It is hoped that the results presented here will motivate such an investigation. In Figures 3 and 4, the diamond symbols plot P_e' vs. S_0. The correction, based on observing k_0 and k_1 empirically for a single constitutive equation at low S_0, works very well in calibrating the observations from two constitutive equations over a much wider range of shear rates. The calibrated values were calculated by actual recomputation of the pressure difference using $P(x)$, with x computed from eq. (15), in place of P_1. This demonstrates

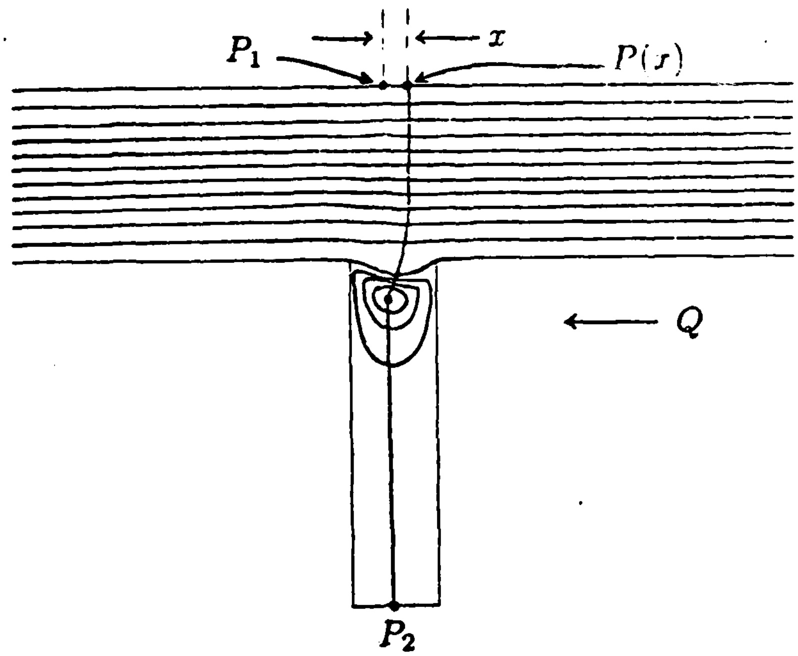

FIGURE 5

Schematic of deformed streamline coordinate system, showing orthogonal curve connecting P_2 location to x; P_1 is located at $x = 0$.

not only that the k-line adjustment computed from low shear-rate data works at high shear rate, but also that the assumptions involved in relating the k-line correction to a modified pressure difference were warrented in the cases presented here.

DISCUSSION

The author believes that the effectiveness of the calibration described is better than coincidental, though it is clearly not exact. The P_e/N_1 curve for the CB model has less curvature than the predicted curve, and this cannot be corrected by a linear calibration curve. The calibrated JSW curve tails off before the predition curve does; There are not enough observation points to tell whether the observed curve has a misplaced maximum or fails to have a maximum at all. An important point to be made is that one cannot expect perfect calibration accuracy everywhere. The hypothesis put forward here explaining the success of the calibration suggests it corrects only for the deformation of the streamline coordinate system, but as explained in refs. [1] and [3], there is expected to be an error in prediction due to the history dependence of the constitutive equations, which would imply that — even on a modified integration path — the existence of a simple shearing flow locally does not imply the flow is viscometric. What is hoped is that the calibration

rule presented here has identified the first-order effects in the deviation of original HPBL prediction and numerical observation.

There are at least three issues which are potentially detremental to the hypothesis put forward here. The first is that the new results of Webster [13] are for different constitutive equations than those used here (Maxwell and Oldroyd B), and they seem to imply nearly the same value of C_1 as the CB and JSW imply, but a value of C_2 that is less than one half the value for CB and JSW. There is no a-priori reason to expect that the calibration constants should be the same for all constitutive equations, or that the deformation of the streamline coordinate system should depend on $N_1/2\sigma$ and $TN_1/2\mu(0)$ in a constitutive-invariant way. If the calibration rule is eventually to prove useful as a tool to calibrate devices for the measurement of N_1 in fluids of unkown properties, the rule obviously must be constitutive-invariant. There are many ways out of this dilemma; the simplest would be to provide a convincing argument that Maxwell and Oldroyd B, which are known to be inadequate fluid models, do not produce the correct deformation of the streamline coordinate system. This is speculation, and investigation is called for.

The second issue bothers the author more than the first; either the numerical results presented here are inaccurate, invariant to treatment by finite differences or finite elements, and even more inaccurate when the finite-difference grid is refined, or else the applicability of the Tanner-Pipkin argument is called into question in the flow involved here. The Tanner-Pipkin argument would be called to question even as an asymptotic argument as $D_e \to 0$; the author is as uncomfortable with such a suggestion as he is with the suggestion that there is not numerical convergence of P_e/N_1 at a fixed, low D_e with mesh refinement. Something has to give.

The third and final issue is an intriguing one, and there seem to be so many ways to reconcile what is presented here with the troubling evidence that one hardly knows where to begin. The fact is that the limited experimental results with laboratory pressure measurements do not seem to uncover the deviation of the observed pressure differences from the HPBL prediction which numerical results predict. Numerical results could be wrong — pressure differences are a numerically delicate quantity to predict, and the numerical methods are as yet unanalysed for discretization errors. Experiments could have made errors which compensate for the inaccuracy of the HPBL prediction, and more comprehensive investigations could lead to better numerical/experimental correspondence. It is perhaps more intriguing (and certainly more comfortable) to speculate that real fluids have $C_1 = C_2 = 0$, whereas our attempts at constitutive theories give nonzero values. A final speculation involves the actual measurement procedures; these involve transducers with deformable membranes. The membrane at the wall opposing the slot is so large compared to the width of the slot mouth that, for all practical purposes, the whole upper channel wall is a deformable membrane. It is believed that the effective measurement point for P_1 is the slot centerline, but we should note that the calibrations made here involve the movement of the measurement point of less than $0.32b$ — an extremely small fraction of the length of the transducer — which can make a 50% difference in the hole-pressure!

If the deformable membrane were also subtly deformed, along with the streamline coate system, so as to move the effective measurement point in an upstream direction in the right way, the measurement could be self-calibrating.

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The crossed-triangle macroelement has been identified as an ideal element for non-Newtonian fluid flow calculations involving fluids with integral constitutive equations. In spite of an instability of the pressure approximation, these elements have been shown to have convergent velocities in Stokes flow, and there is strong evidence that pressure-smoothing schemes recover accurate pressures. Earlier studies by the author of Newtonian flow over transverse slots at low Reynolds numbers showed that excellent results could be obtained using the element. Studies of non-Newtonian flows in the same geometry showed

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20. ABSTRACT (continued)

good qualitative agreement with laboratory experiment but led to some puzzling predictions of the pressure difference between the top and bottom of the slot (the "hole-pressure"). In this paper, those puzzling predictions are re-examined, and the deviations from expectation are re-interpreted. They appear to make physical sense and have important ramifications for the calibration of devices which measure the primary normal-stress difference by continuous measurement of the hole-pressure.

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